CHAUVIN® ARNOUX

CHAUVIN ARNOUX GROUP

Multiple representations
of a **three-phase** electrical signal

Qualistar+ power and energy analysers can be used to view the characteristics of a three-phase electrical network instantaneously.

The Qualistar+ models display the signals from all the inputs simultaneously. The measurements are displayed as values, waveforms, spectral representations or Fresnel diagrams.

Time-based representation

Spectral representation

Vectorial representation

Measure up CHAUVIN ARNOUX

Multiple representations of a three-phase electrical signal

1. Three-phase voltages

Electrical energy is transmitted from its production site (source) to the point of distribution (load) via three conductor wires. Industrial installations are the main sites supplied with three-phase alternating current. A three-phase circuit receives three sinusoidal voltages with the same frequency.

A three-phase voltage distribution system (fig. 1) comprises 3 line conductors and (sometimes) what is known as a "neutral" conductor:
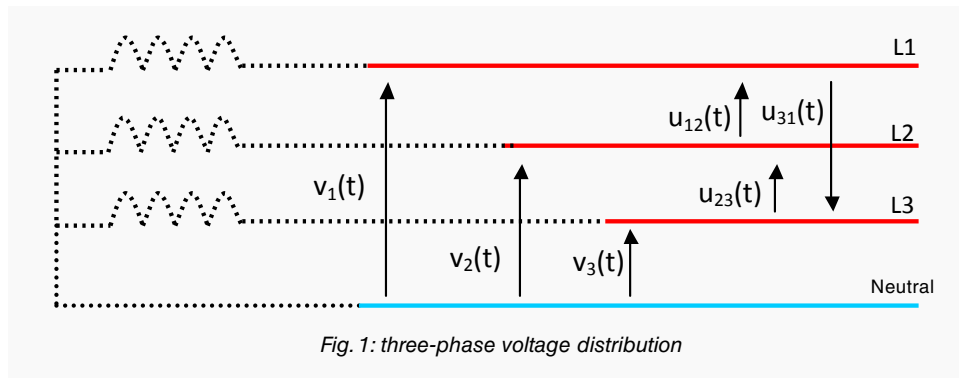

Fig. 1: three-phase voltage distribution

A. Equations and associated properties

The three-phase voltage system represented by v1(t), v2(t) and v3(t) is defined by the following equations:

$$v_1(t) = V_1 \sqrt{2} \, \sin(\omega.t)$$
$$v_2(t) = V_2 \sqrt{2} \, \sin(\omega.t - \frac{2\pi}{3})$$
$$v_3(t) = V_3 \sqrt{2} \, \sin(\omega.t - \frac{4\pi}{3})$$

In theory:
- the amplitudes of the 3 voltages are equal,
- the respective phase angles are equal (120°),
- the voltages are perfectly sinusoidal.

In practice, these properties are not checked. The significance of the faults can be quantified by measuring the **unbalance and total harmonic distortion**.

The voltages v1(t), v2(t) and v3(t) are called "phase-to-neutral" voltages. The voltages measured between phases are called "phase-to-phase" voltages. When three-phase systems of phase-to-neutral voltages are perfect, the equations of these phase-to-neutral voltages are defined as follows:

$$u_{12}(t) = v_1(t) - v_2(t) = V\sqrt{2}\sqrt{3} \, \sin(\omega.t + \frac{\pi}{6})$$
$$u_{23}(t) = v_2(t) - v_3(t) = V\sqrt{2}\sqrt{3} \, \sin(\omega.t - \frac{\pi}{2})$$
$$u_{31}(t) = v_3(t) - v_1(t) = V\sqrt{2}\sqrt{3} \, \sin(\omega.t + \frac{5\pi}{6})$$

The amplitudes (and the RMS values) of the phase-to-phase voltages are √3 greater than those of the phase-to-neutral voltages. The sum of the 3 components of a three-phase voltage system is equal to 0.

Multiple representations of a three-phase electrical signal

B. Time-based representation

The three-phase system of phase-to-neutral voltages (fig. 2) comprises 3 voltage sinusoids which follow one another with an offset of 6.6 ms. Indeed,

$$\omega.t_0 = \frac{2\pi}{3}$$
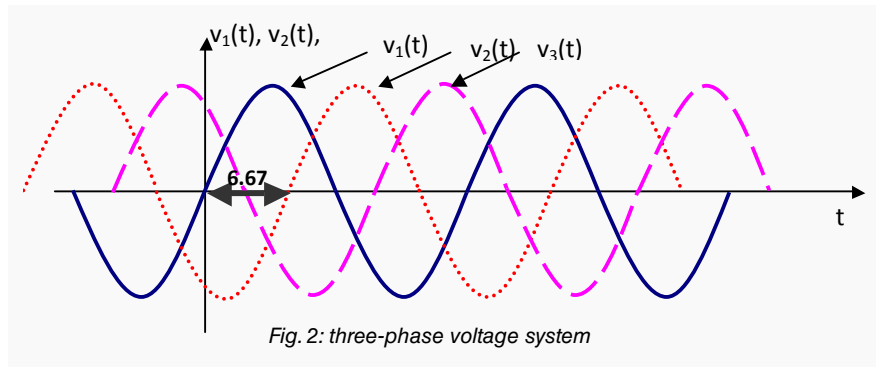
$$t_0 = \frac{2\pi}{3.\omega} = \frac{2\pi}{3.314} = 6,67 ms$$

Fig. 2: three-phase voltage system

C. Vectorial representation

The three-phase system of phase-to-neutral voltages described above can be represented in a vectorial diagram (fig. 3). The lengths of the vectors correspond to the amplitudes of the sinusoids which form the system. In electrical engineering, the users are usually interested in the RMS values, so the **vectorial representation** of the system is very often based on the RMS values of the sinusoidal functions.
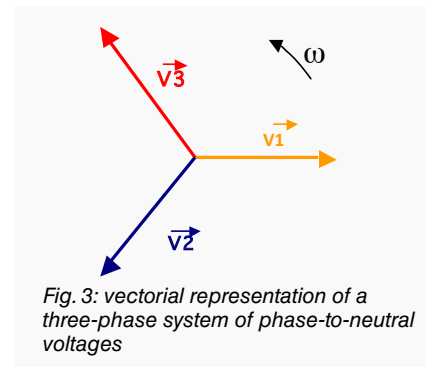

Fig. 3: vectorial representation of a three-phase system of phase-to-neutral voltages

2. Time-based representation of signals

The electrical signals are represented on an oscillogram. A signal is the variation of an analogue electrical quantity (voltage or current) as a function of time. These functions vary continuously over time according to a mathematical law.

A voltage or current signal (fig. 4) which evolves as a function of time can be characterized by a mathematical relation of the following type:

$$t \rightarrow x(t)$$

where x(t) represents the signal's value for each value of passing time t. This is usually called the **instantaneous value**.
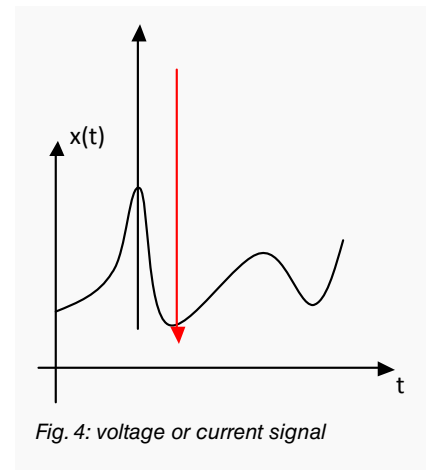

Fig. 4: voltage or current signal

Specific properties

Periodic signal

A signal x(t) is **periodic** when the following relation is true:

$$x(t+T) = x(t)$$

The signal repeats identically over time. The time interval between two moments when the signal shows exactly the same characteristics is called the **T period** (fig. 5).
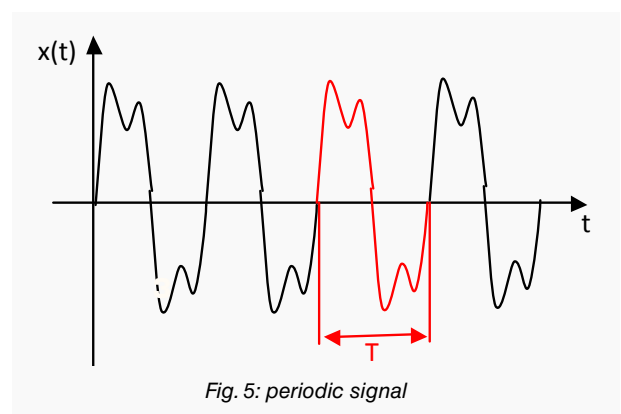

Fig. 5: periodic signal

Multiple representations of a three-phase electrical signal

FOURIER series

When the signal is periodic but non-sinusoidal, and subject to certain mathematical properties (which are generally true for the signals usually dealt with in electrical engineering), a **FOURIER series transform** can be used to obtain a time-based representation comprising only a DC signal and sinusoidal signals whose frequencies are multiples of the basic signal's frequency.

This property is particularly useful for reasons of calculation (calculation with complex numbers) and representation (**spectral representation**).

This transform is performed as follows:

Let x(t) be a periodic signal with the period T.

The decomposition of x(t) into FOURIER series is given by the following formula:

$$x(t) = \begin{cases} X_0 + \\ A_1 \cos \omega t + A_2 \cos 2\omega t + \ldots + A_n \cos n\omega t \quad + \\ B_1 \sin \omega t + B_2 \sin 2\omega t + \ldots + B_n \sin n\omega t \end{cases}$$

where:

$$X_0 = \frac{1}{T} \int_{-\frac{T}{2}}^{+\frac{T}{2}} x(t)\, dt$$

$$A_n = \frac{2}{T} \int_{-\frac{T}{2}}^{+\frac{T}{2}} x(t).\cos(n.\omega..t)\, dt$$

$$B_n = \frac{2}{T} \int_{-\frac{T}{2}}^{+\frac{T}{2}} x(t).\sin(n.\omega..t)\, dt$$

X_0 is called the DC component of the signal x(t);

A_n and B_n are coefficients which represent the amplitudes of the harmonics of order n of the signal x(t).

Examples

DC voltage

$u(t) = E$

The voltage u(t) (fig. 6) does not vary over time.
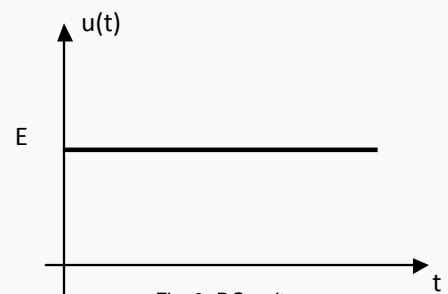It is not periodic, so it cannot be decomposed into a FOURIER series.

Fig. 6: DC voltage

Sinusoidal alternating current (fig. 7)

$i(t) = I_{max} \sin \omega t$

This signal is periodic with period T because:

$$i(t) = I_{max} \sin \omega t$$
$$i(t+T) = I_{max} \sin \left[\omega(t+T) \right] = I_{max} \sin(\omega t + 2\pi) = I_{max} \sin \omega t$$

i(t+T) = i(t) this signal is therefore periodic with period T.
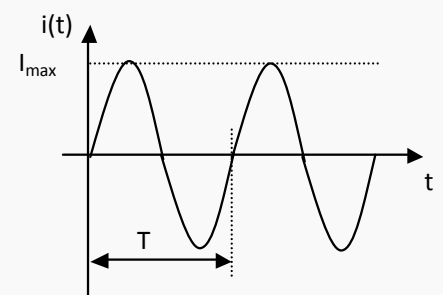There is no point in calculating the FOURIER series of this signal because i(t)
a pure sine curve.

Fig. 7: sinusoidal alternating current

Crenellated current signal (fig. 8)

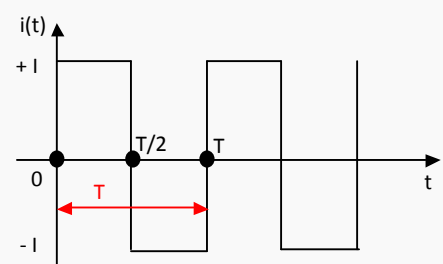$$\begin{cases} i(t) = I \text{ over one half-period} \\ i(t) = -I \text{ over one half-period} \end{cases}$$

Fig. 8: crenelated current signal

Multiple representations of a three-phase electrical signal

❭ *Calculation of the FOURIER series*

Calculation of I_0

$$I_0 = \frac{1}{T}\left[\int_0^{\frac{T}{2}} I\, dt + \int_{\frac{T}{2}}^{T}(-I)\, dt\right] = \frac{I}{T}\left[t_0^{T/2} - t_{T/2}^{T}\right]$$

$$I_0 = \frac{I}{T}\left[\frac{T}{2} - 0 - T + \frac{T}{2}\right] = 0$$

The result of this calculation was expected due to the symmetry in relation to the time axis of the signal i(t).

Calculation of A_n

$$A_n = \frac{2}{T}\left[\int_0^{T/2} I.\cos(n.\frac{2\pi}{T}.t)dt - \int_{T/2}^{T} I.\cos(n.\frac{2\pi}{T}.t)dt\right]$$

$$A_n = \frac{I}{T}.\frac{T}{2\pi.n}\left[\sin(n.\frac{2\pi}{T}.t)_0^{T/2} - \sin(n.\frac{2\pi}{T}.t)_{T/2}^{T}\right]$$

$$A_n = \frac{I}{2\pi.n}\left[\sin(n.\pi) - \sin(0) - \sin(n.2\pi) + \sin(n.\pi)\right]$$

$$A_n = 0$$

Calculation of B_n

$$B_n = \frac{2}{T}\left[\int_0^{T/2} I.\sin(n.\frac{2\pi}{T}.t)dt - \int_{T/2}^{T} I.\sin(n.\frac{2\pi}{T}.t)dt\right]$$

$$B_n = \frac{2.I}{T}.\frac{T}{2\pi.n}\left[-\cos(n.\frac{2\pi}{T}.t)_0^{T/2} + \cos(n.\frac{2\pi}{T}.t)_{T/2}^{T}\right]$$

$$B_n = \frac{I}{\pi.n}\left[-\cos(n.\pi) + \cos(0) + \cos(n.2\pi) - \cos(n.\pi)\right]$$

$$B_n = \frac{I}{\pi.n}\left[-(-1)^n + 1 + 1 - (-1)^n\right]$$

$$B_n = \frac{2.I}{\pi.n}\left[1 - (-1)^n\right]$$

If n is even, B_n is equal to 0. If n is odd, B_n can be written as: $B_n = \frac{4.I}{n\pi}$

The decomposition of the signal i(t) into FOURIER series can then be written as:

$$i(t) = \frac{4.I}{\pi}\sin\omega t + \frac{4.I}{3\pi}\sin 3\omega t + \frac{4.I}{5\pi}\sin 5\omega t + \cdots + \frac{4.I}{n\pi}\sin n\omega t \qquad (n\ odd)$$

Important note:

For calculation of the A_n and B_n values, it may be a good idea to choose the time origin so that it creates symmetries in the mathematical description of the signal. This may save considerable time during the calculation stage.

❭ Time-based representation

With the series calculated previously, it is possible to reconstruct the original signal more or less accurately.

• With the first term only (fig. 9):

$$i(t) = \frac{4.I}{\pi}\sin\omega t$$

This first term is also called the fundamental.

• With the first two terms:

$$(t) = \frac{4.I}{\pi}\sin\omega t + \frac{4.I}{3\pi}\sin 3\omega t$$

• With the first three terms:

$$i(t) = \frac{4.I}{\pi}\sin\omega t + \frac{4.I}{3\pi}\sin 3\omega t + \frac{4.I}{5\pi}\sin 5\omega t$$

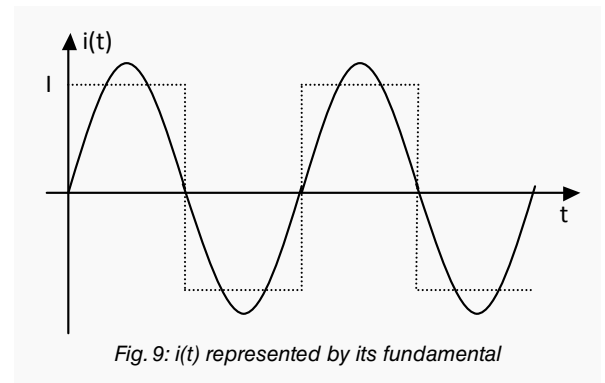The more terms you add in the series, the closer the recomposed signal is to the original signal.

Fig. 9: i(t) represented by its fundamental

3. Vectorial representation of signals

With the Fresnel diagram, we showed the amplitudes and phases of the signals. We can then make use of vectorial operations which are much more convenient than operations on sine and cosine functions.

The vectorial representation of the current-voltage relations in a sinusoidal state is a way of only keeping the signal's phase angle and amplitude. This result can also be achieved by using complex numbers.

A. Temporal - vectorial correspondence

Vectorial representation is only possible for sinusoidal signals.
Let us consider the sinusoidal signal x(t) given by the following relation:

$$x(t) = X \sin(\omega t + \varphi)$$

X is the amplitude of the sinusoidal signal x(t)
ω is the angular frequency of the sinusoidal signal x(t)
φ is the originating phase of the sinusoidal signal x(t)

This representation is based on the correspondence between an amplitude vector X and rotating at the speed ω around a point of origin of this signal O on a time axis (fig. 10).

φ is the originating phase (for t = 0).
The angle travelled by the vector $\vec{X}$ in relation to the original axis Ox is equal to (ωt+φ).
The period T is given by the following relation:

$$T = \frac{2\pi}{\omega}$$

Fig. 10: correspondence between the vector $\vec{X}$ and the signal x(t)
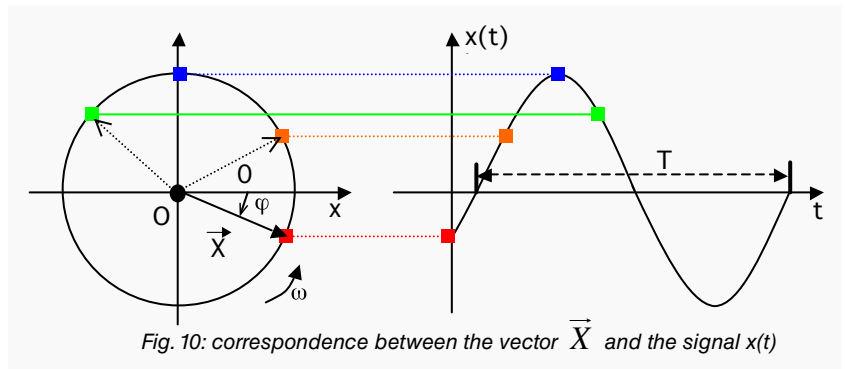
B. FRESNEL diagram

When seeking to study sinusoidal signals (current and voltage) relative to a given circuit, a vectorial representation called a FRESNEL diagram is widely used. The sinusoidal quantities have the same angular frequency. Only the amplitudes and initial phases are different. A representation of the vectors for a given moment is therefore sufficient to deal with the problems (fig. 11).

Usually, the time origin (t = 0) is used.

Fig. 11: circuit (R, L)
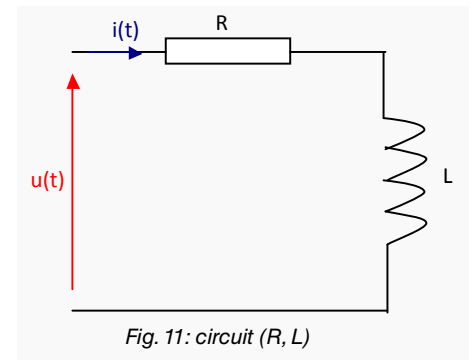
Example 1: inductive circuit

$u(t) = U_{max} \sin \omega.t$ in a steady state, the current i(t) is equal to:

$$i(t) = \frac{U_{MAX}}{\sqrt{R^2 + (L\omega)^2}} \sin(\omega t - \varphi) \qquad \text{with} \qquad tg\varphi = \frac{L\omega}{R}$$

With a FRESNEL diagram (fig. 12), the voltage and current are represented by the vectors $\vec{U}$ and $\vec{I}$:
It should be noted that, conventionally, the angle φ is always oriented from the current towards the voltage.

Fig. 12: vectorial representation
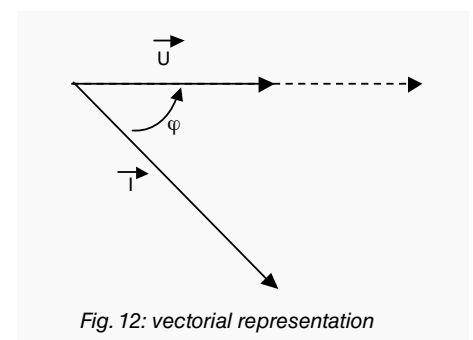
Multiple representations of a three-phase electrical signal

Example 2: three-phase voltage system

Let us take the three-phase voltage system represented by the following equations:

$$v_1(t) = V_{\max} \sin \omega.t$$

$$v_2(t) = V_{\max} \sin(\omega.t - \frac{2.\pi}{3})$$

$$v_3(t) = V_{\max} \sin(\omega.t - \frac{4.\pi}{3})$$

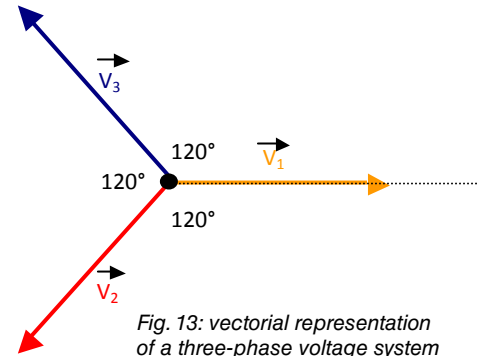Fig. 3 shows the vectorial representation (FRESNEL diagram) of this three-phase system.

Fig. 13: vectorial representation of a three-phase voltage system

4. Spectral representation of signals

A non-sinusoidal signal is more complex. It main contain multiple frequencies. Its spectrum therefore provides information on the different frequency components which it contains. The spectrum of a signal is the representation as a function of the frequencies of the amplitudes of the different components present in the signal.

When a signal x(t) is periodic but non-sinusoidal, and subject to certain mathematical properties (which are generally true for the signals usually involved in electrical engineering), a FOURIER transform can be used to obtain a time-based representation comprising only a DC signal and sinusoidal signals whose frequencies are multiples of the basic signal's frequency.

This property is particularly useful for representing the signal while highlighting the frequencies and amplitudes of the different sinusoidal components given by the calculation of the FOURIER series decomposition.

In the case of a pure sinusoidal signal (fig. 14) described by the following equation:

$$x(t) = X \sin \omega.t$$

we see the appearance of an amplitude X for a sinusoidal signal with an angular frequency ω (or frequency f).

These two pieces of information, which are very important in the context of network analysis, can be plotted on a graph with the amplitude of the sinusoid on the Y axis and the frequency on the X axis. This graphical representation linked to the signal x(t) is the spectral representation of the signal x(t) (fig. 15).
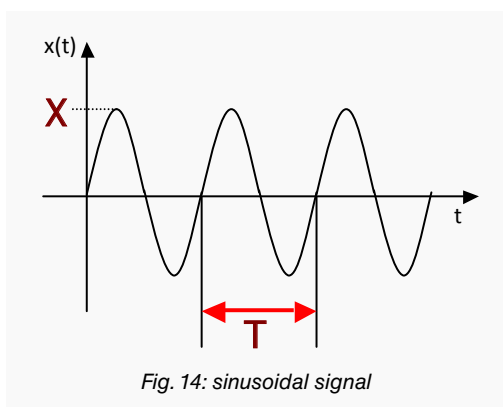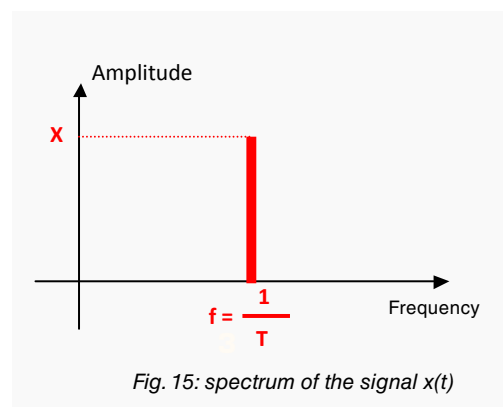

Fig. 14: sinusoidal signal

Fig. 15: spectrum of the signal x(t)

Multiple representations of a three-phase electrical signal

Example: square signal

A current i(t) is described by the following mathematical relation:

$$\begin{cases} i(t) = I \text{ over one half-period} \\ i(t) = -I \text{ over one half-period} \end{cases}$$

Figure 16 shows a time-based representation of i(t).

Calculation of the FOURIER series for this signal i(t) gives the following equation:

$$i(t) = \frac{4.I}{\pi}\sin \omega t + \frac{4.I}{3\pi}\sin 3\omega t + \frac{4.I}{5\pi}\sin 5\omega t + \cdots + \frac{4.I}{n\pi}\sin n\omega t \quad (n \text{ odd})$$

This signal does not have any DC component. It comprises:

- a sinusoidal signal with the frequency f (the frequency of the basic signal) and the amplitude $\dfrac{4.I}{\pi}$

- a sinusoidal signal with the frequency 3f and the amplitude $\dfrac{4.I}{3.\pi}$

- a sinusoidal signal with the frequency 5f and the amplitude $\dfrac{4.I}{5.\pi}$

- etc.

This description can be used to establish the following spectral (or harmonic) representation (fig. 17):
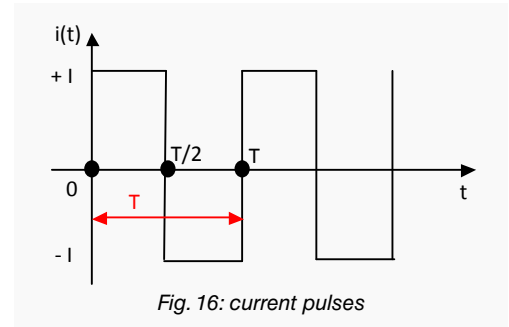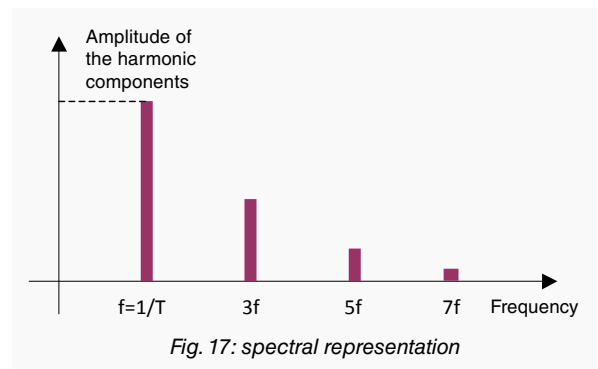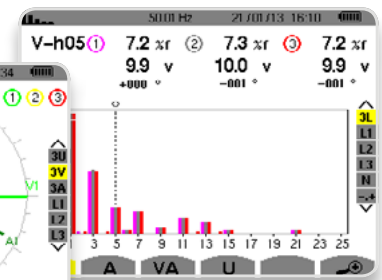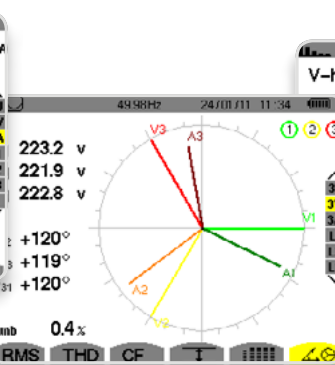

Fig. 16: current pulses

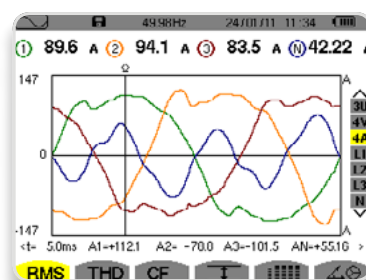Fig. 17: spectral representation

The power and energy analysers in the **Qualistar+ range** can be used to view all these representations.

CHAUVIN ARNOUX
GROUP